Lattice Method for Charged Hadrons in Magnetic Fields

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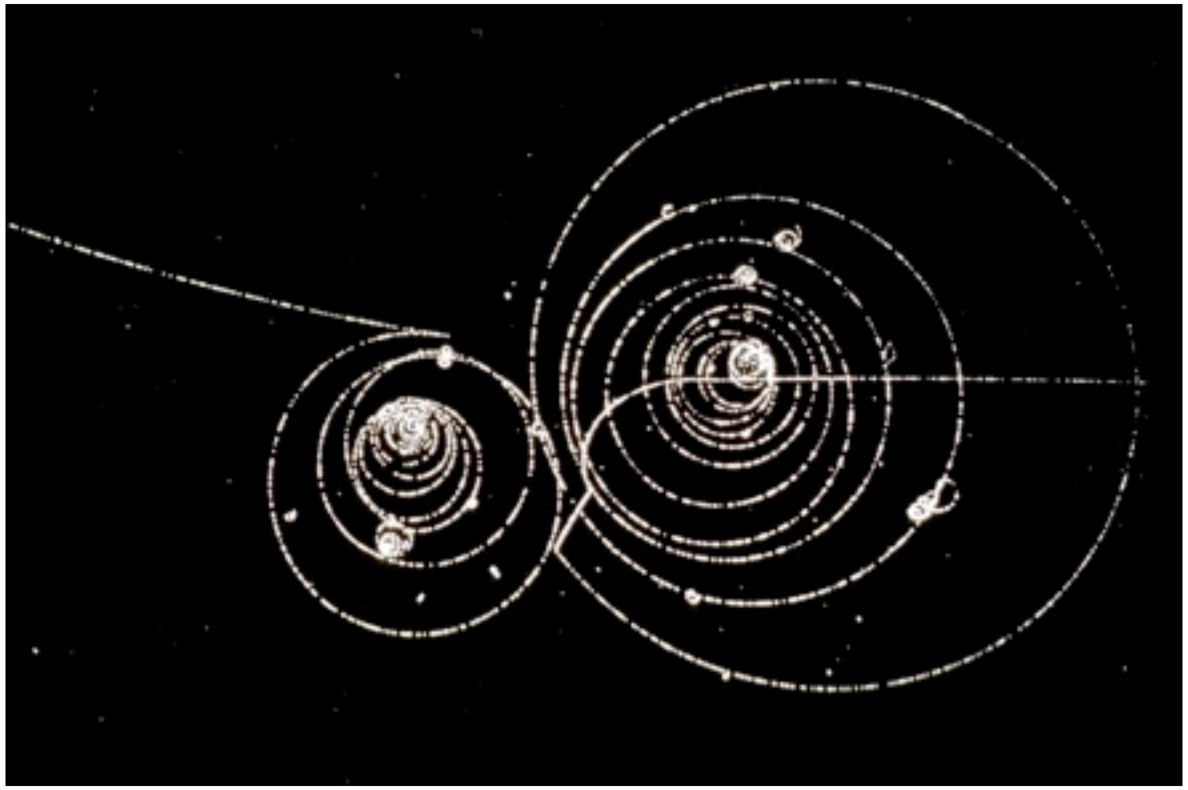
Lattice Method for Charged Hadrons in Magnetic Fields

[Tiburzi, Vayl PRD (2013)]

- **Goal:**
  Determination hadron properties from *Lattice QCD*



- Hadrons: nucleon to light nuclei
- Properties: electromagnetic to start

Magnetic observables to compute

Magnetic polarizability of pion

$$\Delta H = -\frac{1}{2}\beta_M \vec{B}^2$$

- ChPT vs. Experiment: 2.5σ discrepancy

  [Large cast of characters...]

- Will **COMPASS** resolve?
- Contribution to hadronic light-by-light (constraint in π loop models)

  [Engel, Patel, Ramsey-Musolf PRD (2012)]

Magnetic polarizability of nucleon

- Experiment: 50% - 100% uncertainty
- ChPT in single and few nucleon systems

  [Large cast of characters...]

- Dominant error in determining nucleon EM mass splitting

  [Walker-Loud, Carlson, Miller PRL (2012)]

- Help constrain unknowns in proton structure corrections to μ-H

  [Hill, Paz PRL (2011)]

Magnetic moments and polarizabilities of light nuclei

- Little known about moments of Λ hypernuclei $\,^5_\Lambda\mathrm{He}$ $\,^7_\Lambda\mathrm{He}$

Challenging observables to compute

Magnetic polarizability of pion

$$\Delta H = -\frac{1}{2}\beta_M \vec{B}^2$$

$$T^{\mu\nu}(k',k) = \int_{x,y} e^{ik\cdot y - ik'\cdot x}\langle H|T\Big\{J^\mu(x)J^\nu(y)\Big\}|H\rangle$$

Compton Tensor: currently beyond reach of lattice QCD

Magnetic polarizability of nucleon

Signal $$\sum_{\{A_\mu\}} \langle qqq(t)\overline{qqq}(0)\rangle \sim e^{-Mt}$$

Noise^2 $$\sum_{\{A_\mu\}} \langle qqq(t)\overline{qqq}(t)qqq(0)\overline{qqq}(0)\rangle \sim e^{-3m_\pi t}$$

Signal/Noise

$$\sim e^{-(M-\frac{3}{2}m_\pi)t}$$

Baryons are statistically noisy.... scales exponentially with A

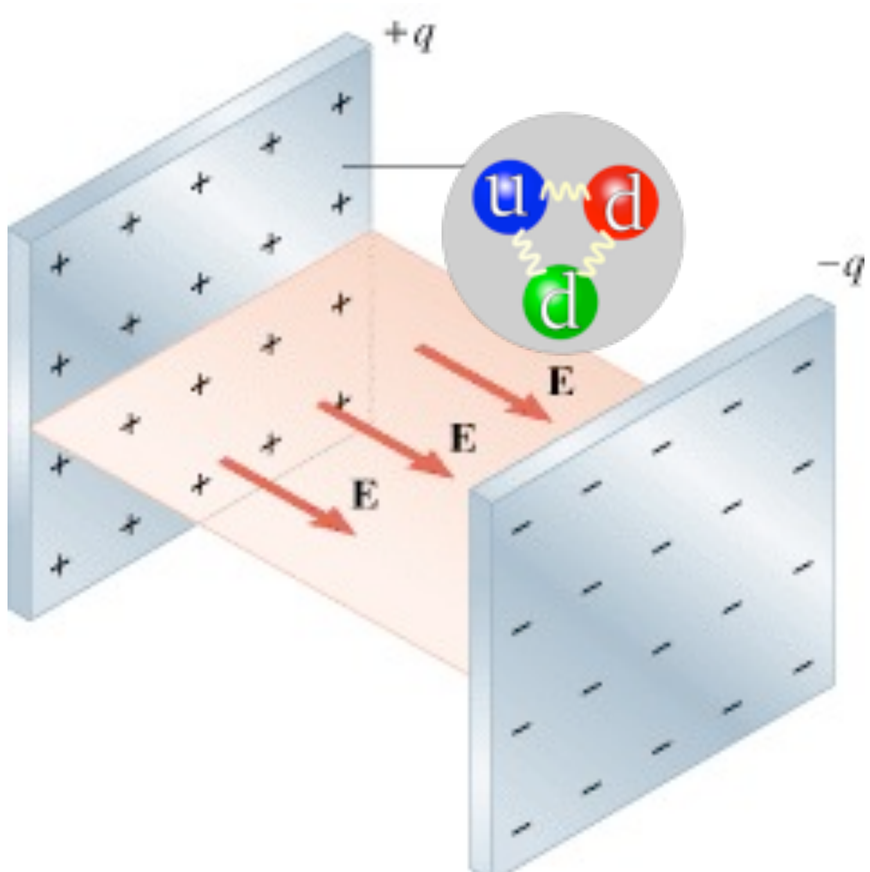
Magnetic moments and polarizabilities of light nuclei

Single current insertion with B>1 never tried

Lattice QCD in External Fields

Couple classical electromagnetic fields to quarks and then study hadron spectroscopy

$$D_\mu = \partial_\mu + ig\,G_\mu + iqA_\mu$$



Gauge links

$$U_\mu(x) = e^{igG_\mu(x)} \in SU(3)$$

$$U_\mu^{\text{e.m.}}(x) = e^{iqA_\mu(x)} \in U(1)$$

Strong magnetic field studies on thermodynamic lattices

[Chernodub, et al.]

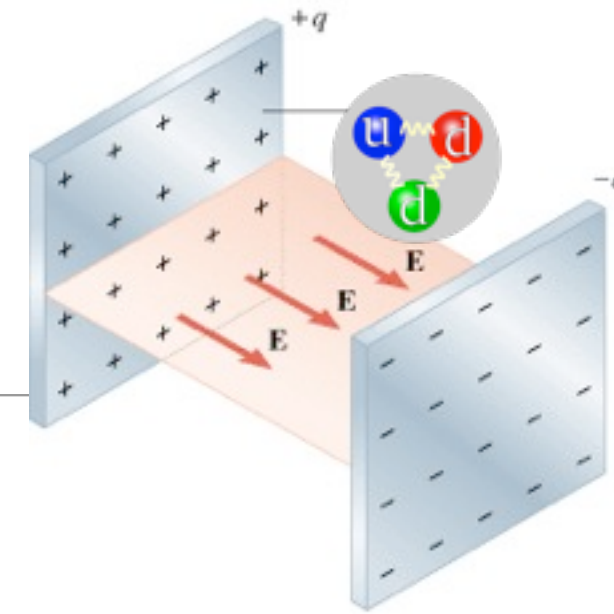
[D'Elia, Mukherjee, et al.]

[BM&W collaboration]

Exploratory *weak* electric field studies: U(1) field couples only to valence quarks

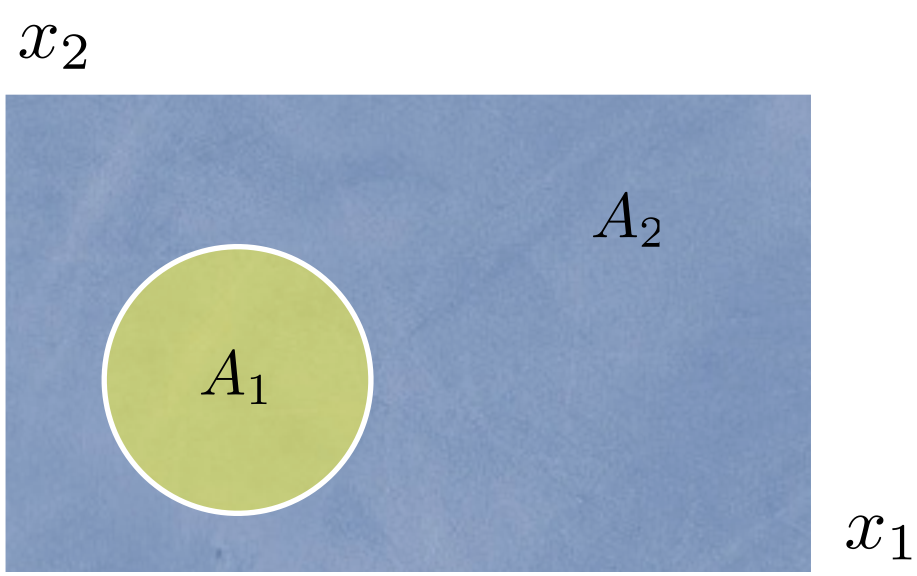
[Detmold, Tiburzi, Walker-Loud]

Lattice QCD in External Fields



Couple classical electromagnetic fields to quarks ...

Magnetic field $\vec{B} = B\hat{x}_3$ Electric field $\vec{\mathcal{E}} = \mathcal{E}\hat{x}_3$



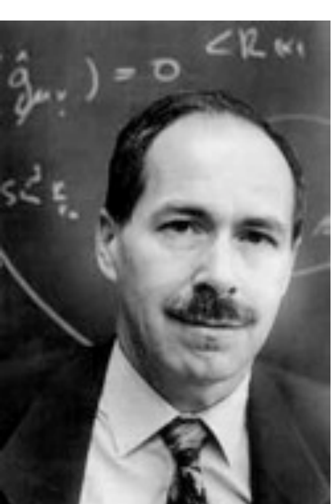
Gauge links

$$U_\mu^{\text{e.m.}}(x) = e^{iqA_\mu(x)} \in U(1)$$

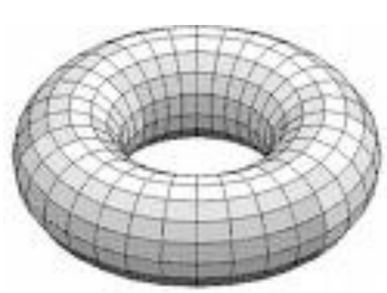
't Hooft quantization

$$qB = \frac{2\pi n}{L^2}$$

$$q\,\mathcal{E} = \frac{2\pi n}{\beta L}$$



Torus $$e^{iqBA_1} = e^{-iqBA_2}$$

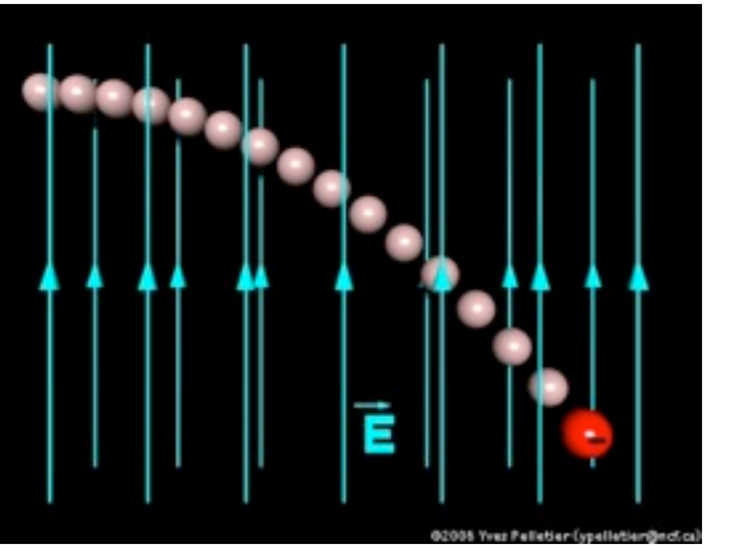


$$A_1 + A_2 = L_1 L_2$$

Thanks:
Taku & Urs!

... and then study hadron spectroscopy

Lattice QCD in Electric Fields



Method basics are basic

- Measure hadronic correlation functions in classical electromagnetic fields
- Study field strength dependence to determine parameters in effective action

E.g. *charged* pion in electric field

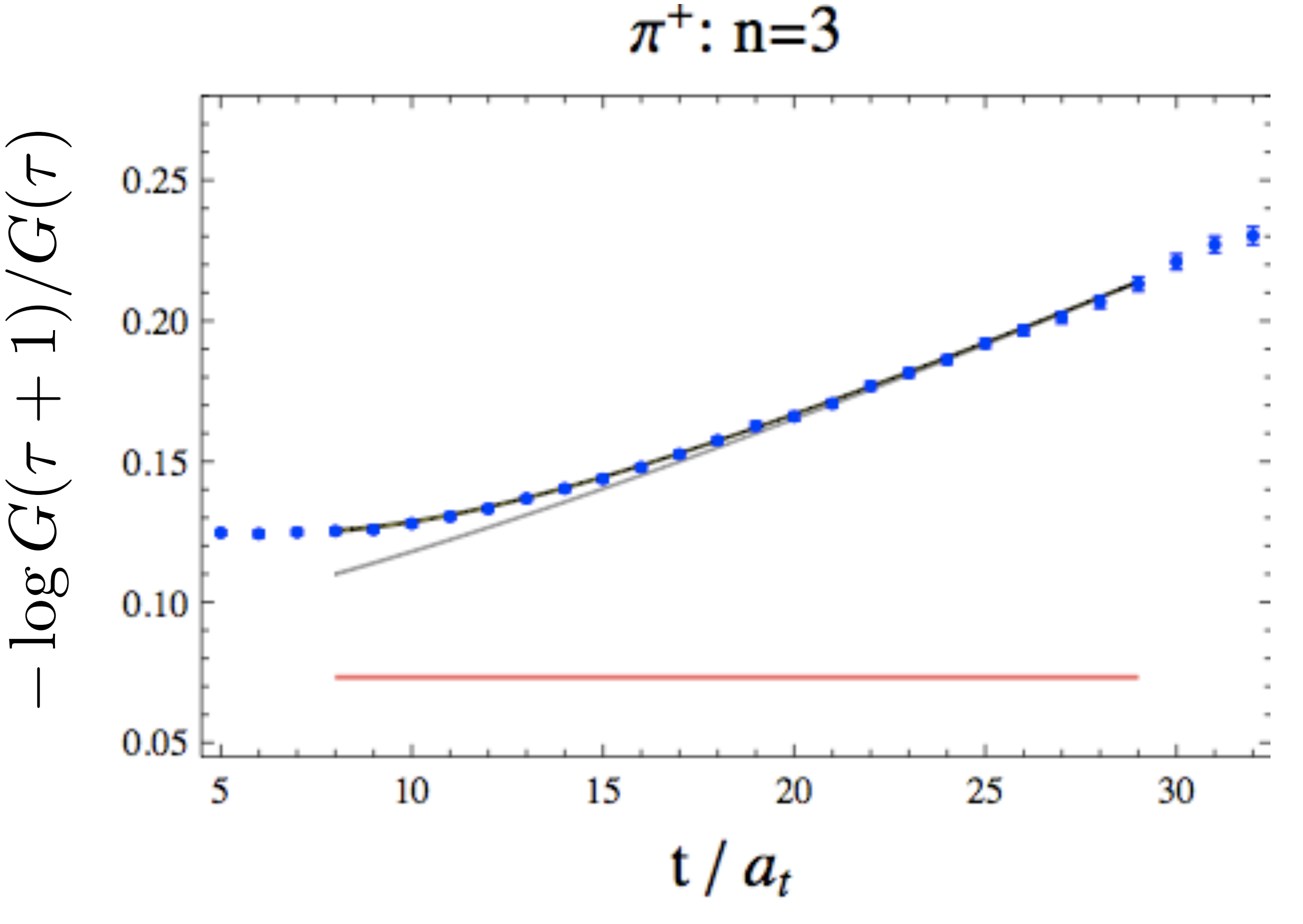
$$A_\mu = -\mathcal{E} x_4 \delta_{\mu 3}$$

[Detmold, Tiburzi, Walker-Loud PRD (2009)]

Anisotropic clover lattices (HadSpec)

$20^3 \times 128 \qquad m_\pi = 390\,\mathtt{MeV}$

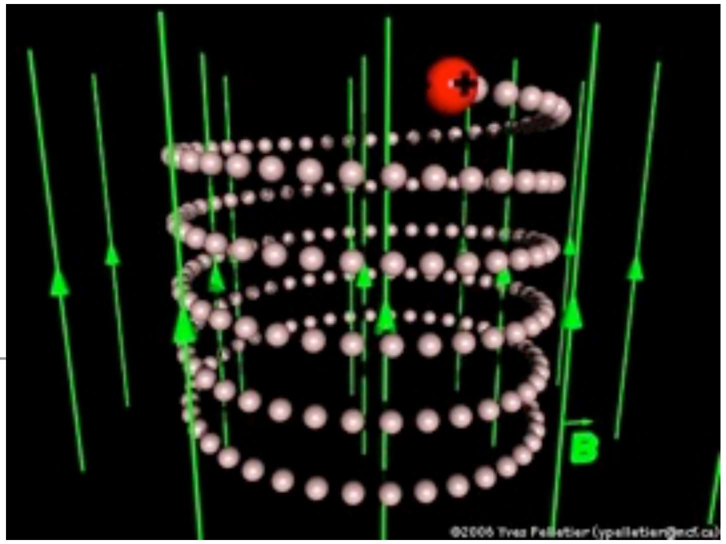
$$E = m_\pi + \frac{1}{2}\alpha_E \mathcal{E}^2 + \ldots$$



$$G(\tau) = \langle \tau | \frac{1}{2\mathcal{H} + E^2} | 0 \rangle = \frac{1}{2}\int_0^\infty ds\, e^{-\frac{1}{2} s E^2} \langle \tau | e^{-s\mathcal{H}} | 0 \rangle$$

[Schwinger PR (1951)]
[Tiburzi NuPhA (2008)]

Lattice QCD in Magnetic Fields



Method basics are basic

- Measure hadronic correlation functions in classical electromagnetic fields
- Study field strength dependence to determine parameters in effective action

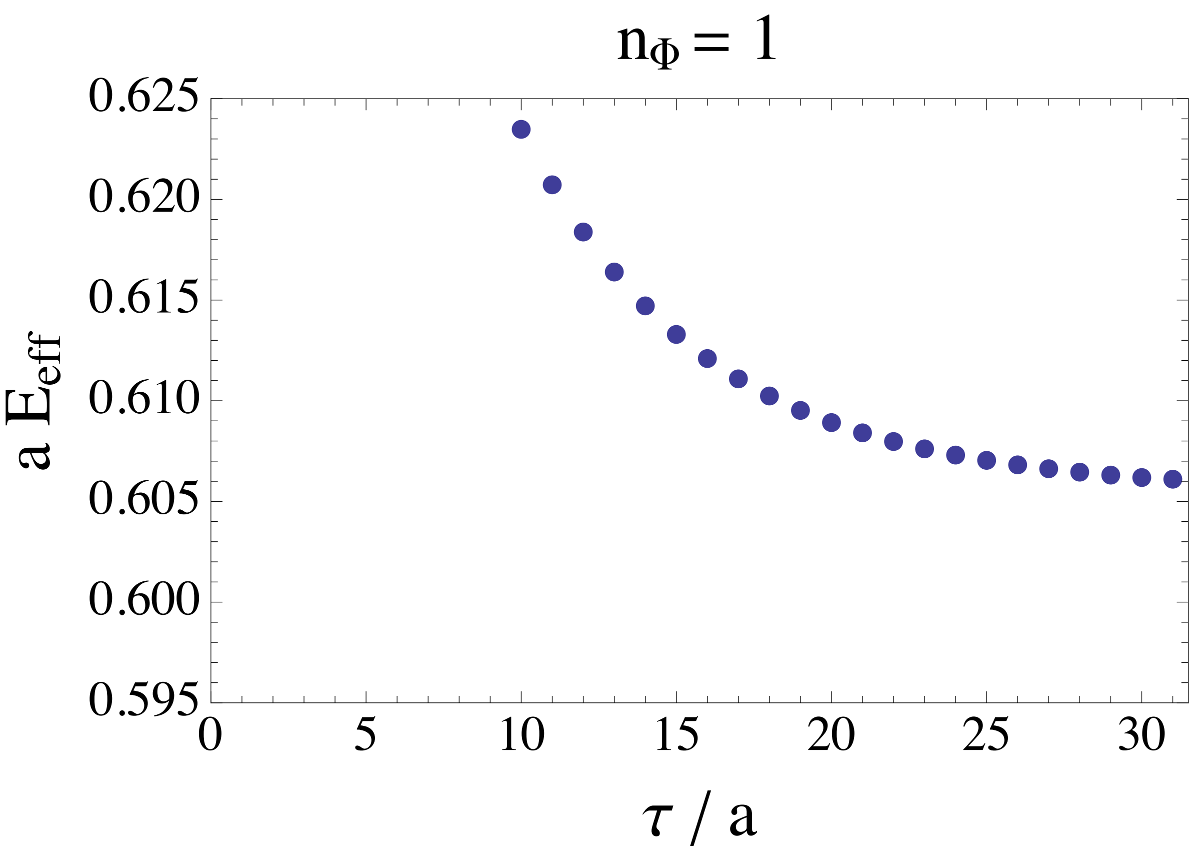
E.g. *charged* scalar in magnetic field

$$A_\mu = -B x_2 \delta_{\mu 1}$$

Synthetic data for heavy scalar "nucleon"

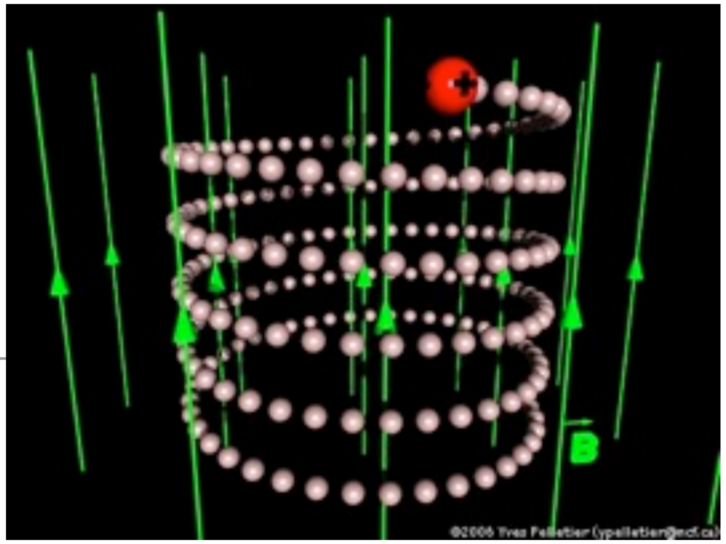
$32^3 \times 64$

$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x}, \tau) \chi^\dagger(\vec{0}, 0) \rangle$$



Need to measure small energy shifts

Lattice QCD in Magnetic Fields



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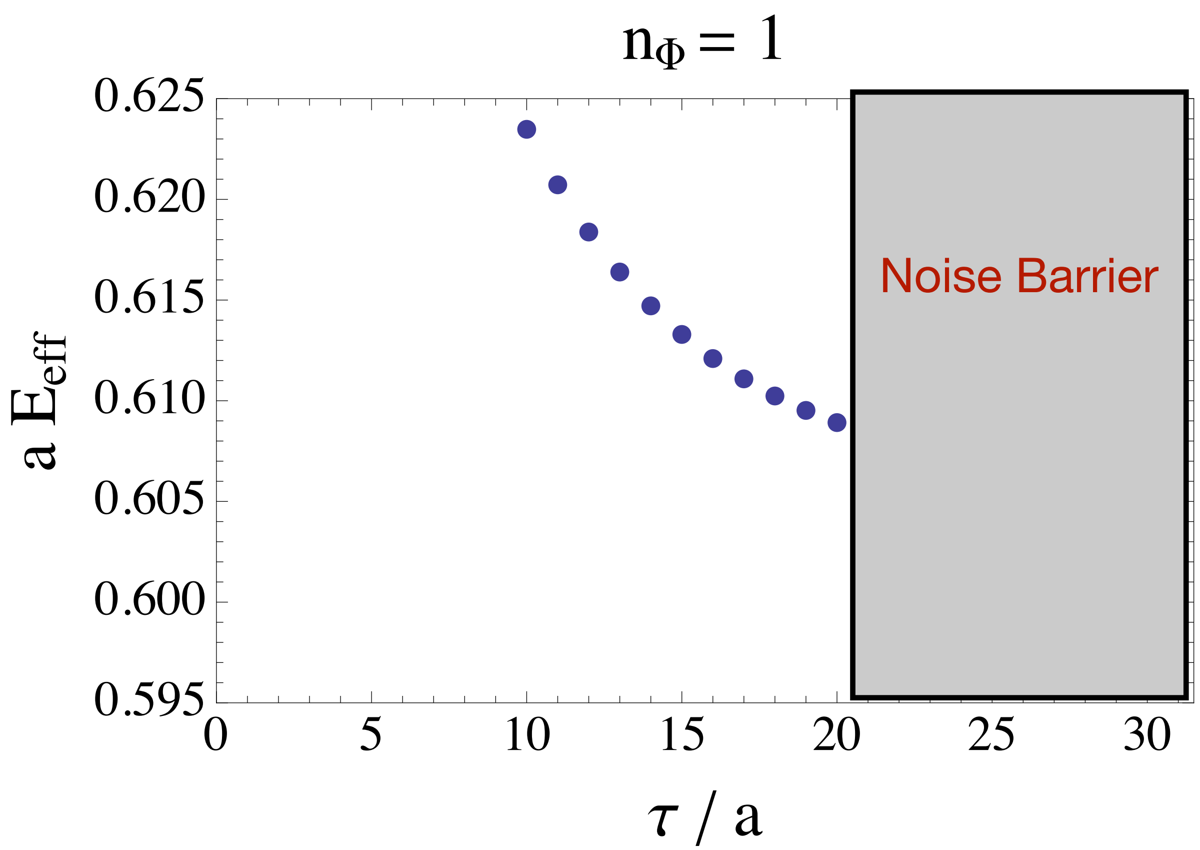

Need to measure small energy shifts

This model does not have hadronic excited states

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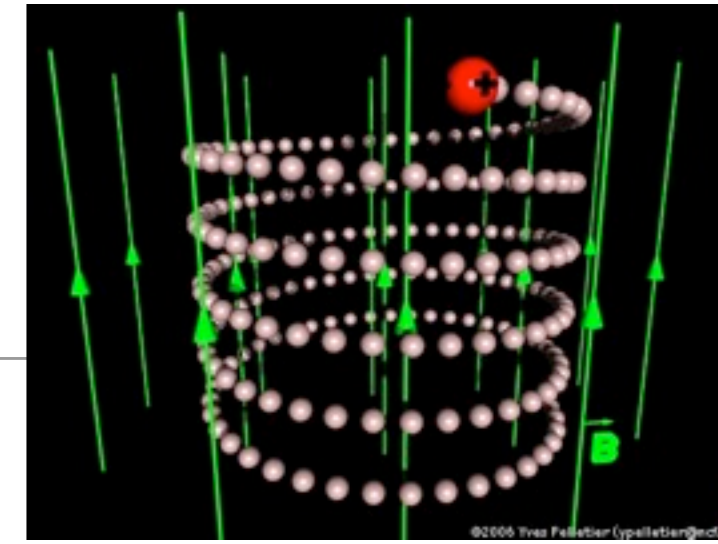

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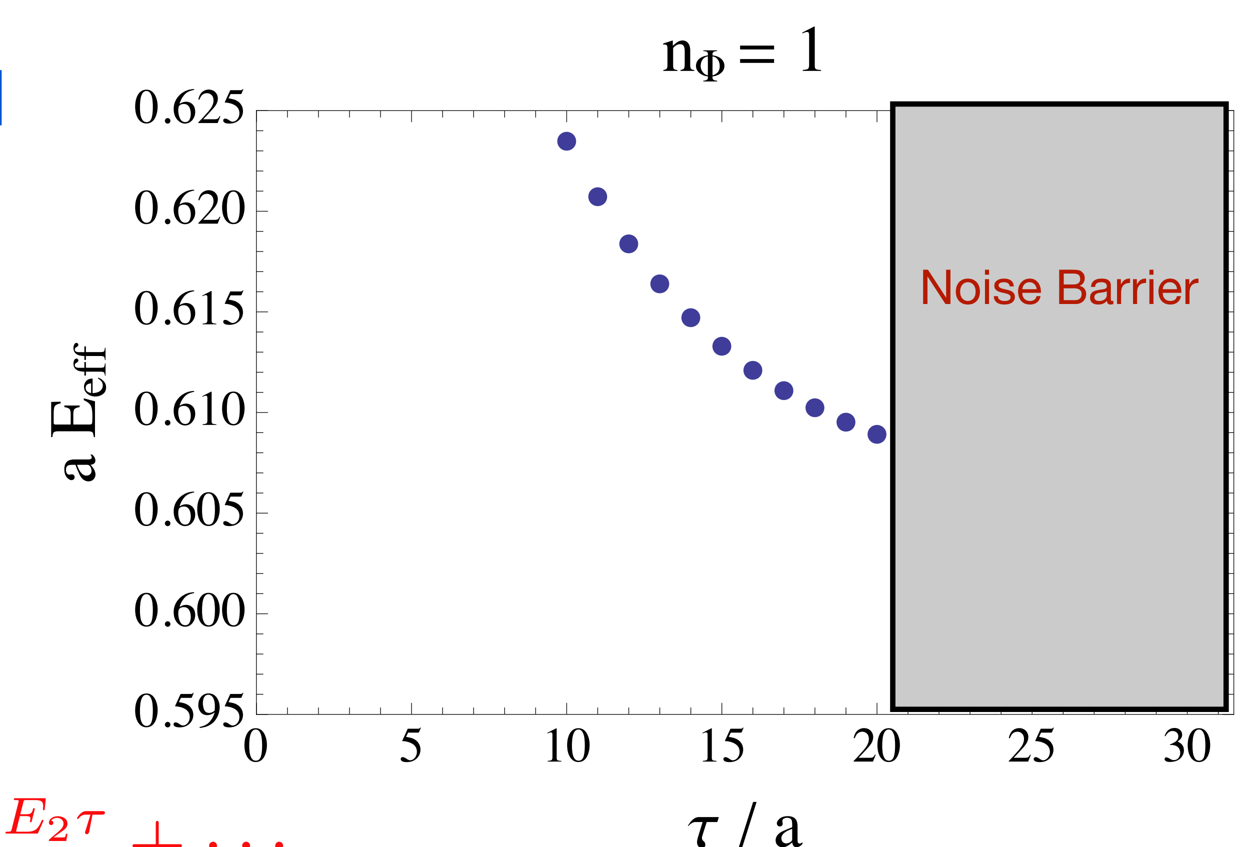
$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x}, \tau) \chi^\dagger(\vec{0}, 0) \rangle$$

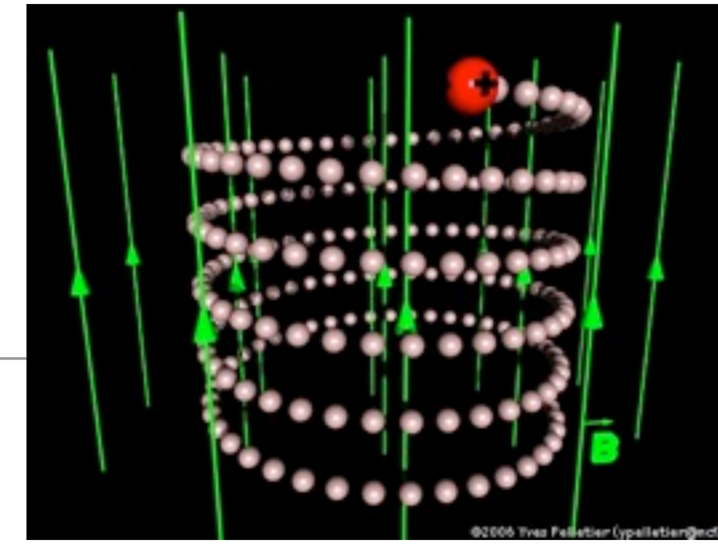
$$= Z_0 e^{-E_0 \tau} + Z_1 e^{-E_1 \tau} + Z_2 e^{-E_2 \tau} + \cdots$$



This model does not have hadronic excited states

... but there are Landau levels

“Wisdom” on the subject



fitting to the two smallest fields.

For charged particles, there is the possibility of Landau levels on the order of $|qB/(2M)|$ in the presence of magnetic fields, where q and M are the charge and mass of the particle, respectively. It is a linear term that is not eliminated by the averaging procedure. Their effects only show up at very large times, larger than where we fit the data. We

[Names withheld, JOURNAL (YEAR)]

$$A_\mu = -Bx_2\delta_{\mu 1}$$

Noise Barrier

$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x},\tau)\chi^\dagger(\vec{0},0)\rangle$$

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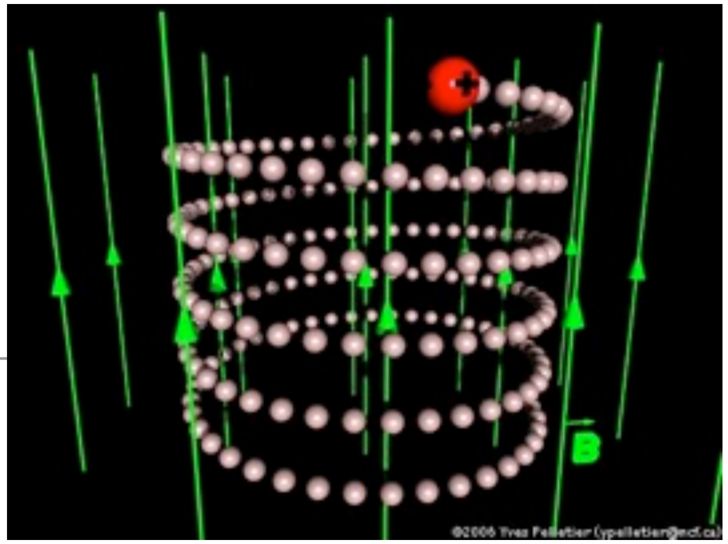
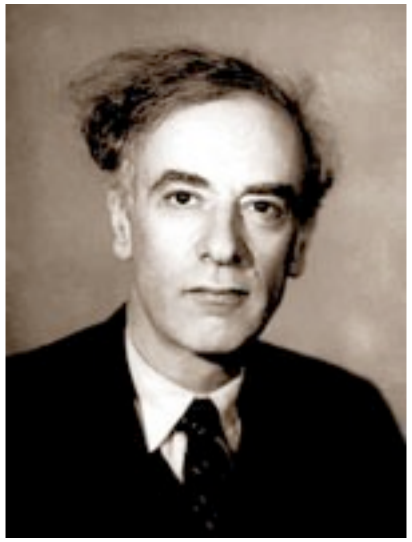
Charges in Magnetic Fields



- Quantization condition restrictive $qB = \dfrac{2\pi n}{L^2}$
  Ideally $|QB| \ll M^2$

- Charged particles: Landau levels $E_n = |QB|\left(n + \dfrac{1}{2}\right)$



$$\Delta E_n / M^2 = |QB|/M^2$$

Desired physics: deal with pile up

- Lattice two-point correlation function $A_\mu = -B x_2 \delta_{\mu 1}$

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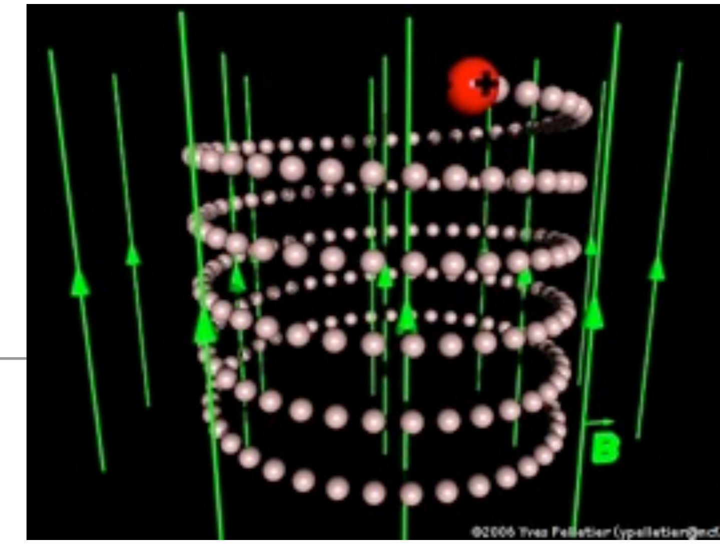

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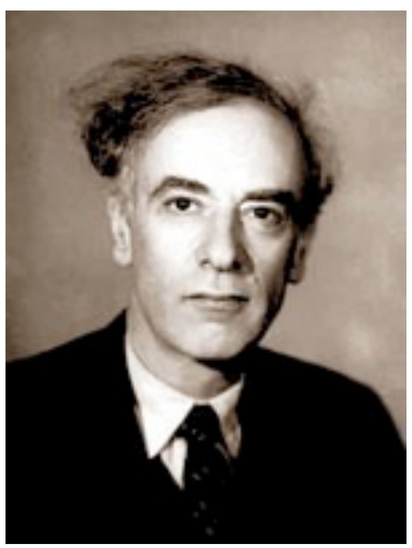

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Desired physics leads to pile up!

- Lattice two-point correlation function $\quad A_\mu = -Bx_2\delta_{\mu 1}$

$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x},\tau)\chi^\dagger(\vec{0},0)\rangle$$

$$= \int_0^\infty ds \frac{e^{-\frac{1}{2s}\left(\tau^2 + s^2 \mathcal{M}_B^2\right)}}{\sqrt{s\cosh(QBs)}}$$

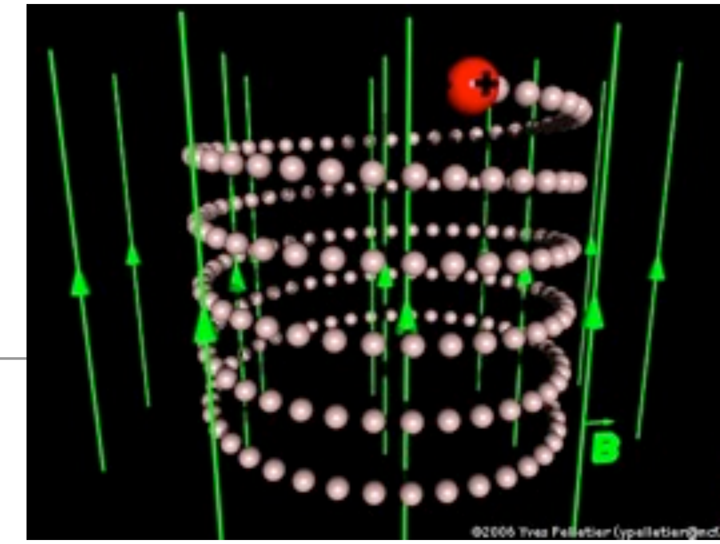
À LA SCHWINGER

Sum all Landau levels

1). Complicated function to fit

2). Landau levels subject to finite-volume effects

Charges in Magnetic Fields



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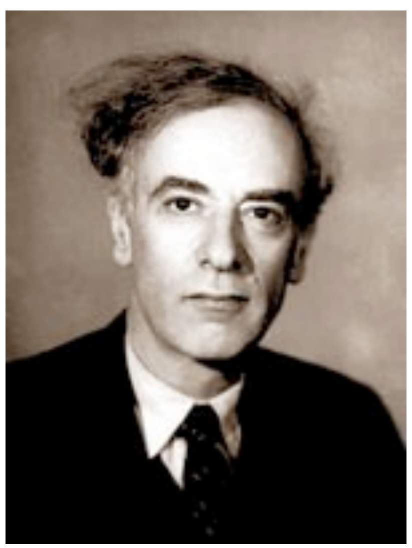

$$\Delta E_n / M^2 = |QB|/M^2$$ Desired physics leads to pile up!

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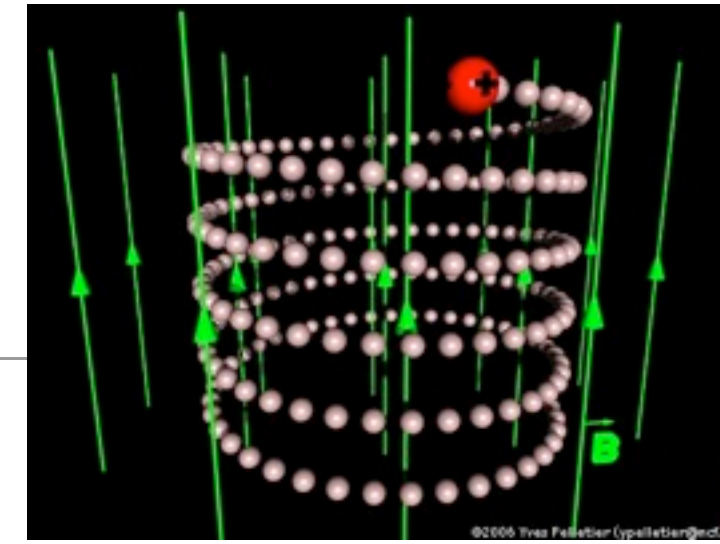

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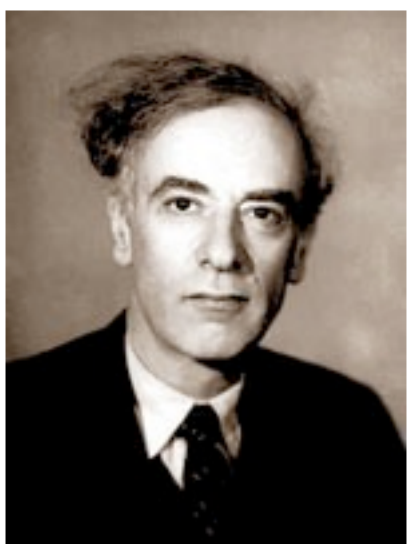


$$\Delta E_n / M^2 = |QB|/M^2$$ Desired physics leads to pile up!

- Lattice two-point correlation function $A_\mu = -Bx_2\delta_{\mu 1}$

$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x},\tau)\chi^\dagger(\vec{0},0)\rangle \times \psi^*_{n=0}(x_2)$$

$$= e^{-E_0 \tau}$$

À LA SCHWINGER

Project out lowest Landau level

1). Simple exponential function to fit

2). Lowest level has finite-volume & discretization effects

Discretization Effects $\psi_{n=0}(x_2)$

- Largest corrections in strongest fields $x_2 \lesssim \frac{1}{\sqrt{|QB|}}$

- Weak fields suspect perturbation about continuum is OK

Assume $$-\frac{1}{a^2}\sum_{j=1}^{3}[\delta_{\vec{n}+\hat{j},\vec{n}'}U_{j,\vec{n}} + \delta_{\vec{n},\vec{n}'+\hat{j}}U^{\dagger}_{j,\vec{n}'} - 2\delta_{\vec{n},\vec{n}'}]$$

Can make precise with Symanzik analysis

$$\Rightarrow \quad T + V = \frac{4}{a^2}[\sin^2(a\hat{p}_2/2) + \sin^2(eaB\hat{x}_2/2)]$$

Expand in powers of field

$$\boxed{V_{2j} \propto \frac{1}{a^2}(eaBx_2)^{2j+2} \lesssim b^j V_0}$$

Rayleigh-Schrödinger Perturbation Theory!

$$\Delta H = -\frac{C_1}{12a^2}[(a\hat{p}_2)^4 + b^4(\hat{x}_2/a)^4]$$

$$\psi_{n=0}(x_2) = \psi^{(0)}_{n=0}(x_2) + \frac{b}{16\sqrt{6}}\psi_{n=4}(x_2)$$

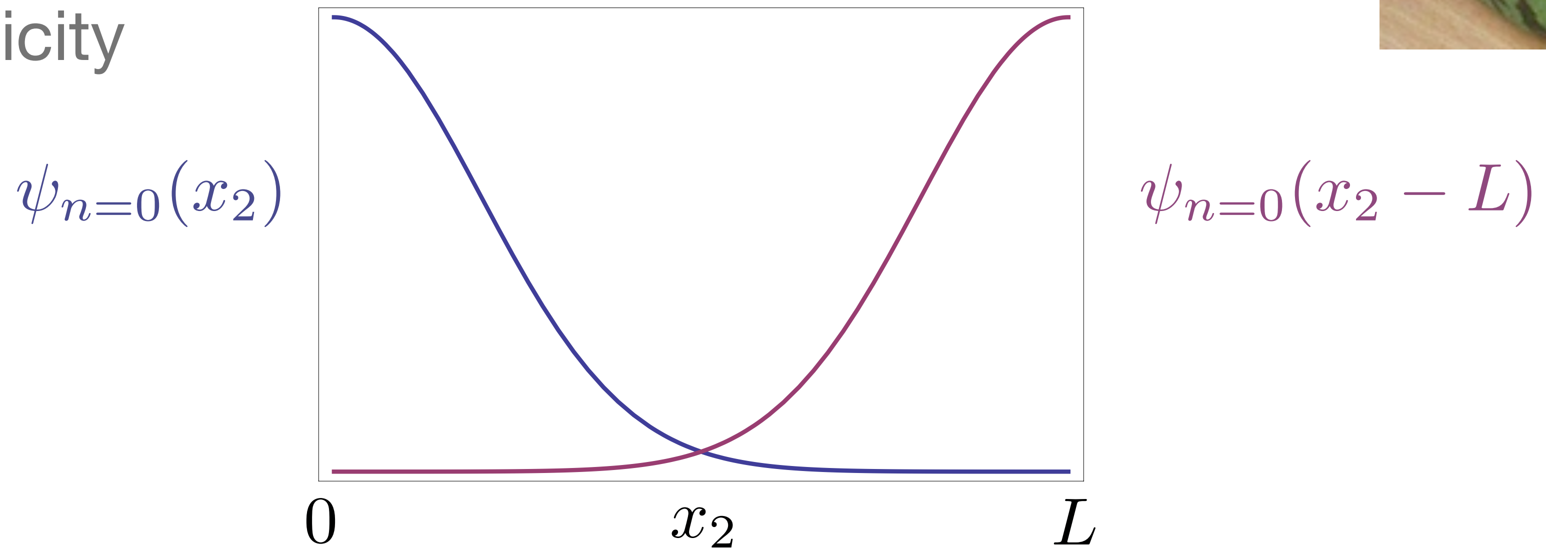
Energy of lowest lattice Landau level

$$a^2E_0^2 = a^2M^2 + |b| - \left(\frac{C_1}{8} + \beta\right)b^2 + \mathcal{O}(b^3)$$

+ Tiny

Finite Volume Effects

- Naïve periodicity



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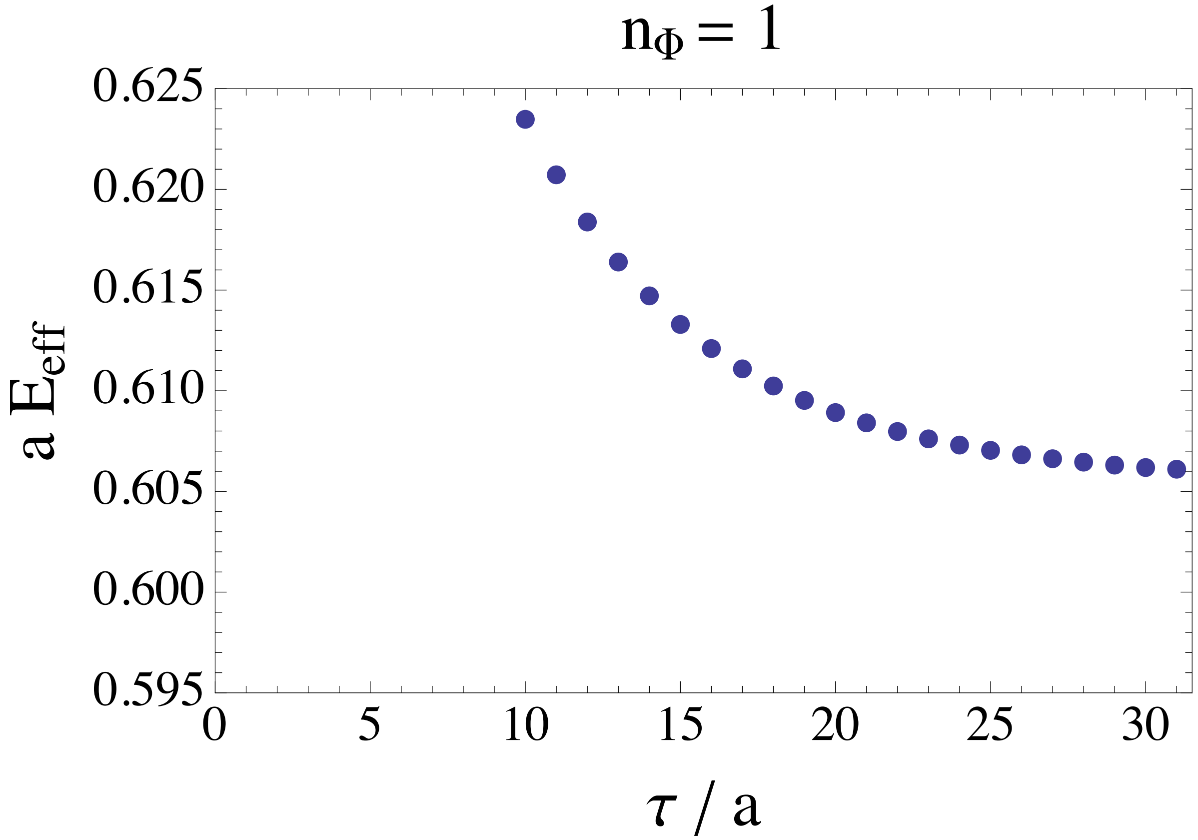

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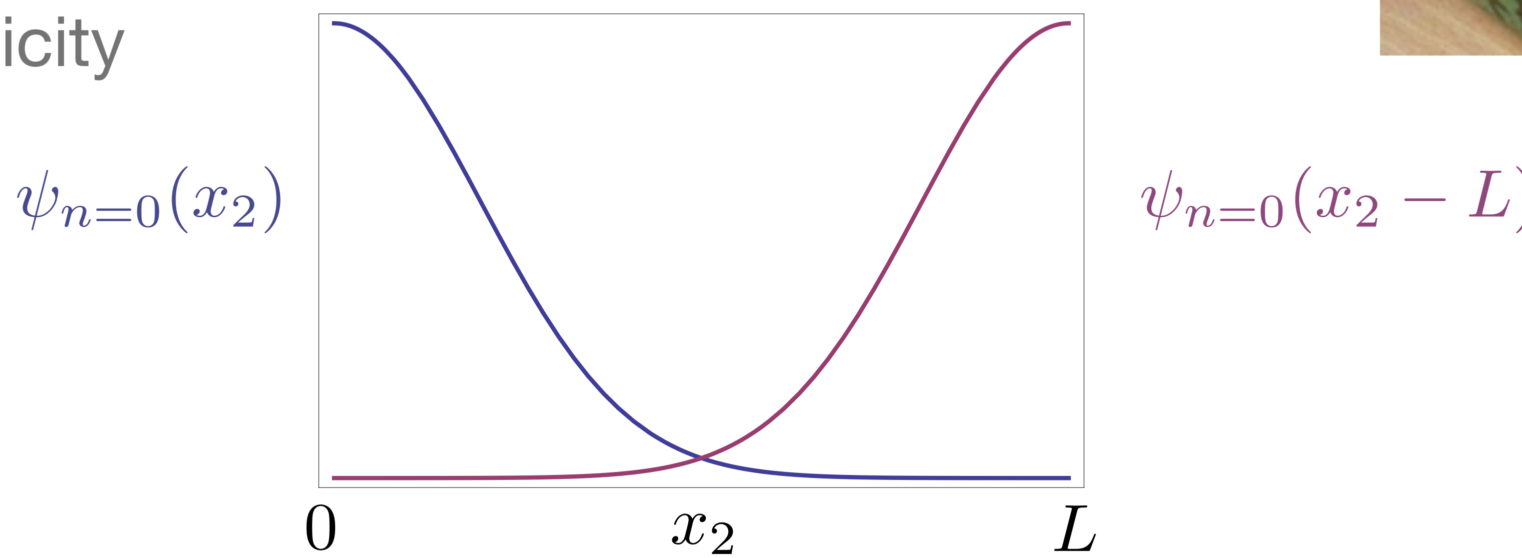
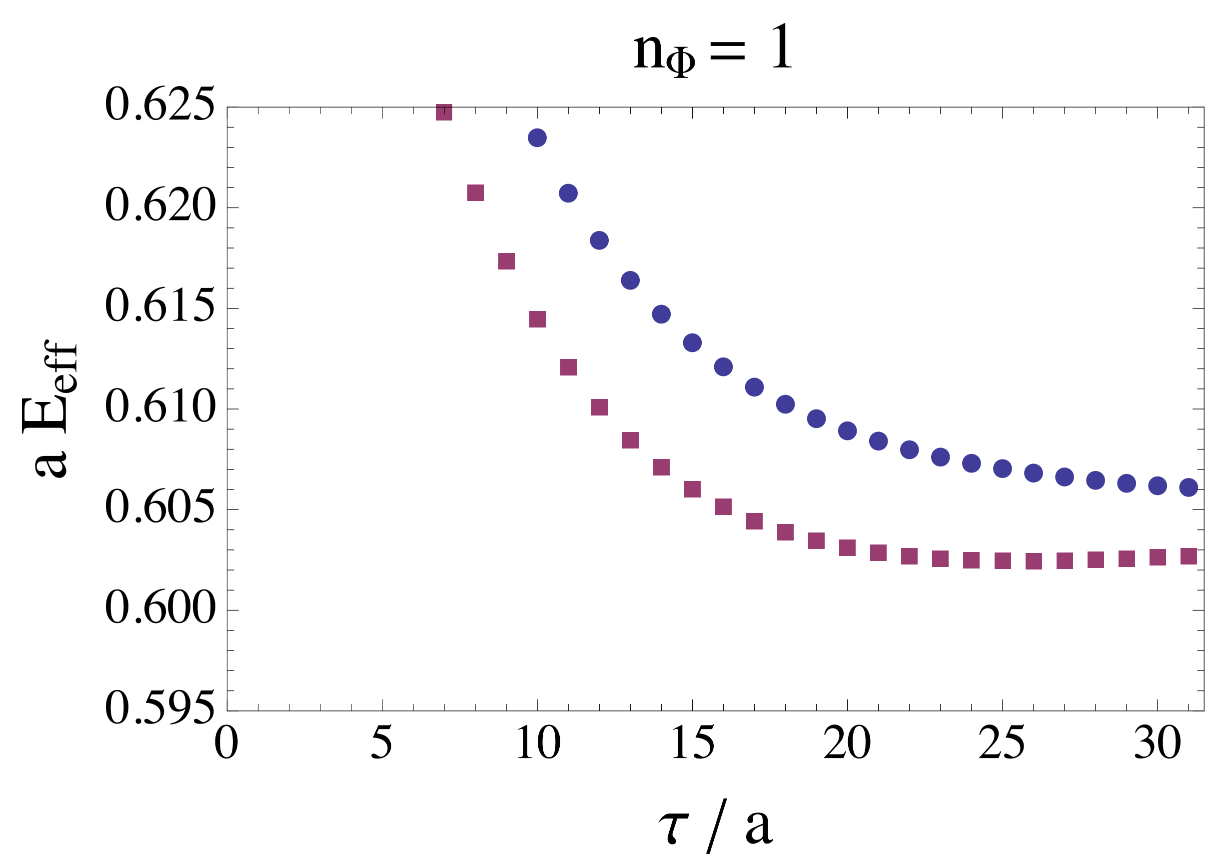


$$G(\tau) = \sum_{\vec{x}} \langle \chi(\vec{x}, \tau) \chi^\dagger(\vec{0}, 0) \rangle$$

$$\times \left[\psi_{n=0}(x_2) + \psi_{n=0}(x_2 - L) \right]$$



Naïve is, well, naïve

Finite Volume Effects

Harmonic oscillator is ***not*** translationally invariant

- **Magnetic translation**: effect of translation eaten up by gauge freedom (∞ degeneracy)

- **Magnetic periodicity**: discrete subgroup on torus (finite degeneracy)

$$\phi(x + L\hat{x}_2) = e^{iQBLx_1}\phi(x)$$

[Al-Hashimi, Wiese Ann. Phys. (2009)]

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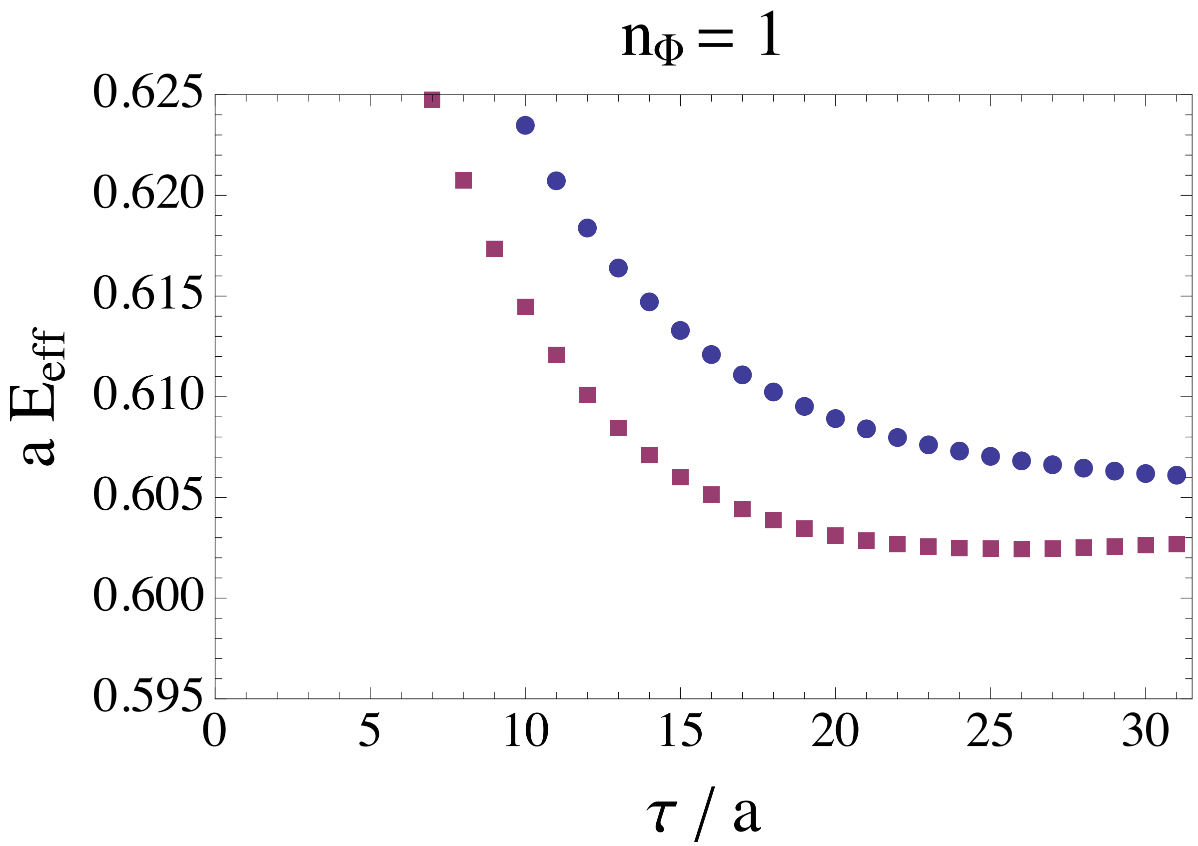

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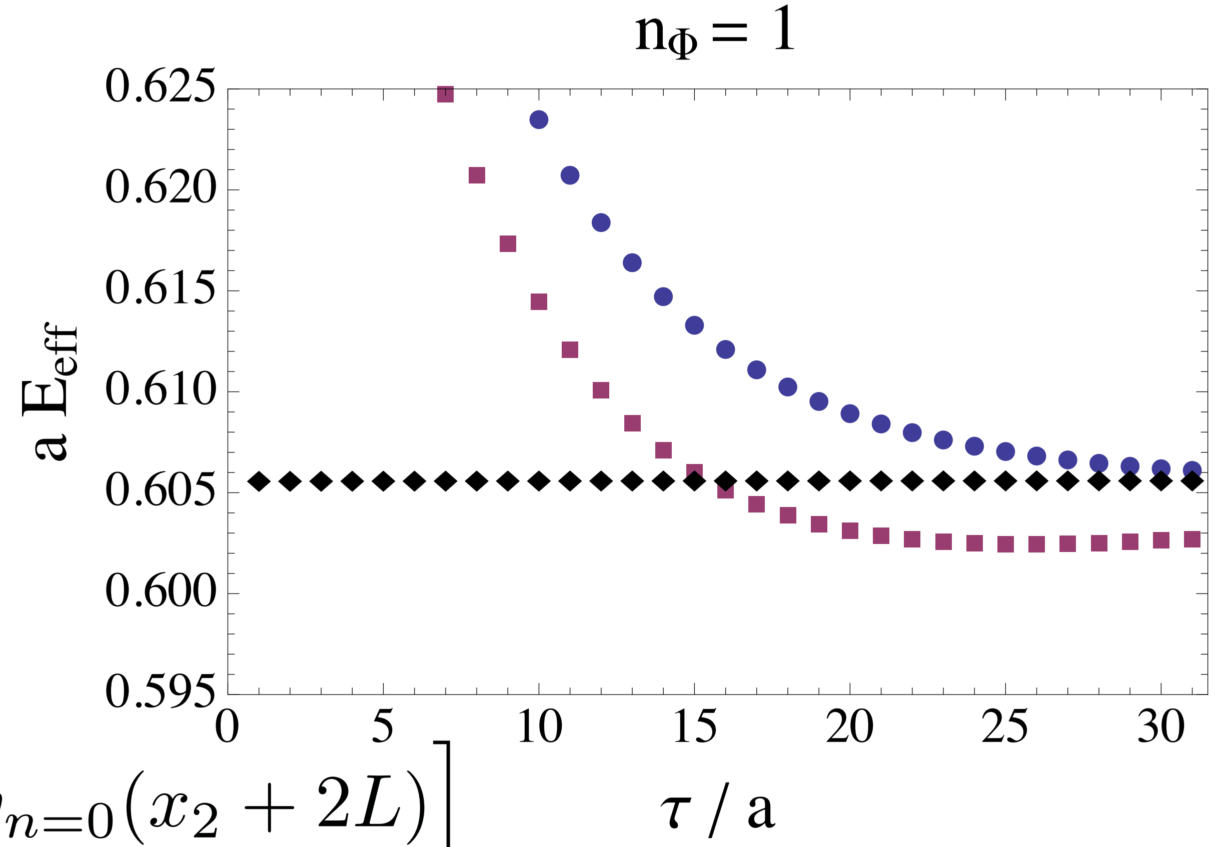
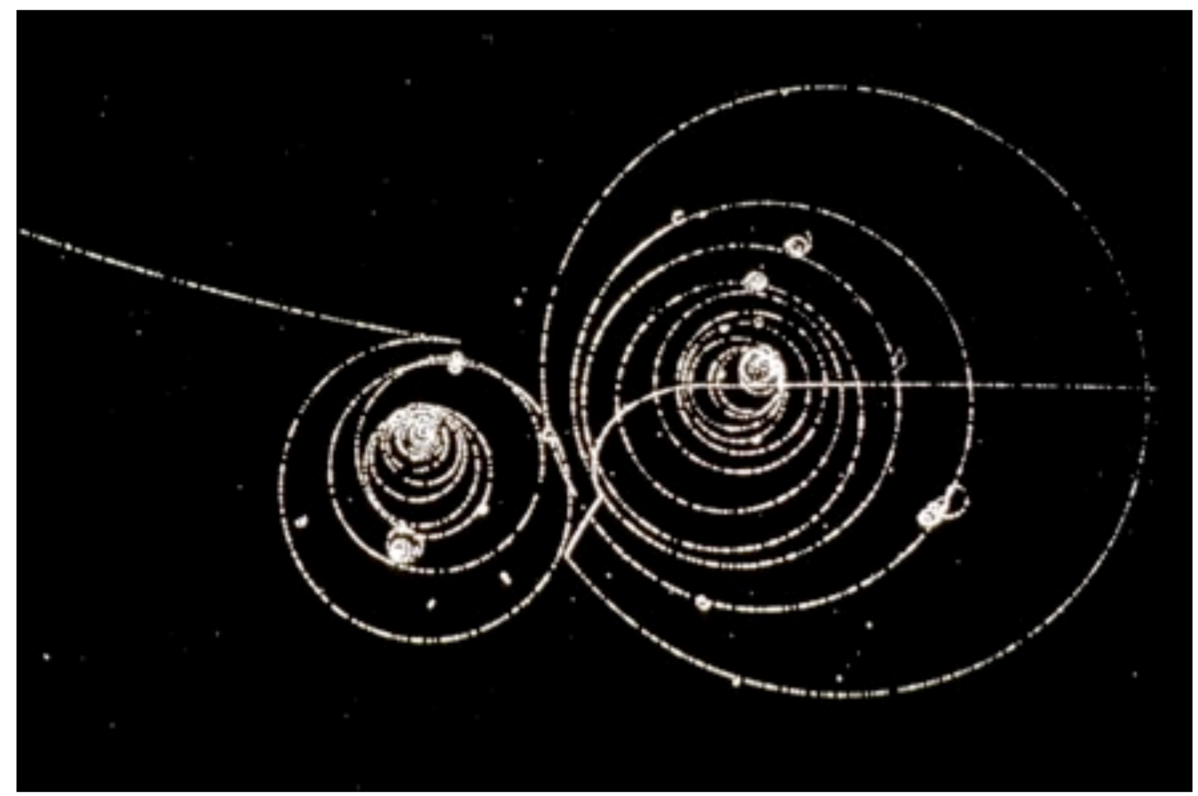
$$\times \Big[\psi_{n=0}(x_2) + e^{-iQBLx_1}\psi_{n=0}(x_2 - L)$$
$$+ e^{iQBLx_1}\psi_{n=0}(x_2 + L) + e^{-2iQBLx_1}\psi_{n=0}(x_2 + 2L)\Big]$$



Magnetic Method for Charged Hadrons

- Landau levels pile up for physically interesting case of small magnetic fields



- Larger the hadron mass, the more the pile up. Nuclei are charged!
- Projection of lowest lattice Landau level possible: discretization primarily affects energy, magnetic periodicity
- Scalar case treated in detail: π He-4 Most nuclei have spin . . .